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Dynamical systems theory applied to short walking trials

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ABSTRACT

Human walking is an extremely complex neuromuscular activity whose simplicity disappears when an attempt is made to provide a quantitative description of the process. The dynamical systems theory provides a framework for analyzing the stability and chaotic nature of dynamical systems, employing Floquet multipliers (FM) and long and short-term Lyapunov exponents (LE), respectively. This report compares FM and LE from three methods: method A (false nearest neighbors and numerical approximation), method B (false nearest neighbors and semi-analytical technique) and method C (singular value decomposition and semi-analytical technique). Data from 33 healthy older adults with no history of falls were used to explain the dynamic system. A surrogate center of mass trajectory was calculated for the analysis of sway in the transverse plane. Results revealed methodological differences in LE and FM calculations with semi-analytical solutions providing closer approximations to observed gait behavior. The long-term LE from Methods A and B were similar, but other LE pairings differed. Method A's short-term LE indicated chaotic gaits for all subjects, while long-term LE from Methods A and B indicated chaos for half the subjects. Method C showed non-chaotic gait for most subjects. Method B's FM indicated over 30% of subjects had unstable gait. Method C yielded values of LE and FM that most closely matched the subjects' gait patterns. This study offers a methodological foundation for gait analysis using short time-series data, facilitating deeper insights into both stability and chaos within gait dynamics.

1. Introduction

Healthy human gait requires moving the center of mass (CoM) through 3D space in a stable manner (Bruijn and Van Dieën, 2018). The stability of walking gait has been studied using dynamical systems theory (DST) (Bruijn et al., 2013). Floquet theory (FT), a part of DST, has been applied to both healthy and abnormal gait (Bhat et al., 2021; Dingwell et al., 2001) (Lockhart and Liu, 2008; Reynard et al., 2014). DST enables the analysis of the stability and the chaotic nature of a given dynamical system using the Floquet multipliers (FM) and the Lyapunov exponents (LE), respectively (Greiner, 2010; Teschl, 2012). A system is considered stable if the FM lie within the unit circle, i.e., FM magnitude ≤ 1 (Teschl, 2012). The LE defines the rate of separation of infinitesimally close trajectories upon being perturbed, such that if LE$>$0, the system is deemed to be chaotic (Greiner, 2010).

The common way FT is used to study gait is to use the FM to study its stability (Hurmuzlu and Basdogan, 1994) or the LE to study the divergence of the curves using Rosenstein et al.'s algorithm (Rosenstein et al., 1993) or Wolf et al.'s algorithm (Wolf et al., 1985). Rosenstein's and Wolf's algorithms use numerical approximations to calculate the maximum LE using the limited amount of collected data (Bruijn et al., 2013), but do not comment on the values of FM. These studies analyze the chaotic nature of gait (LE) without commenting on the stability measure (FM). Further, these techniques can induce approximation errors into the values of the exponents and multipliers. Studying stability as well as the chaotic nature of the gait using semi-analytical techniques (SAT) would result in lower approximation errors with results closer to the real-world gait dynamics. Hence, a SAT approach to study both the stability and the chaotic behavior of gait, that corresponds to real-world observations, is needed.

FT requires a large amount of continuous data for statistical precision (Bruijn et al., 2009). Such an analysis for several neurological conditions as well injuries is difficult due the patient's inability to walk for an extended duration. Sloot et al. studied the chaotic nature of gait using multiple shorter datasets (Sloot et al., 2011), but did not comment on the stability of the gait. Creating a larger dataset by stitching shorter datasets can induce noise in the system (Sloot et al., 2011). Singular value decomposition (SVD) has been used to reduce the amount of noise present in a system and to reduce its dimensionality (Sadasivan and Dutt, 1996). Hence, the use of SVD should be explored in the context of

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shorter walking trials.

The goal of this report is to determine whether approximation errors induced using the Rosenstein algorithm are eliminated using the SAT and SVD approach, and if the resulting values of LE and FM correspond to real-world observations.

2. Methods

2.1. Data collection and processing

Adults over the age of 50 years are often studied due to their higher risk of fall induced injury (Bergland and Wyller, 2004; Painter et al., 2009). Therefore, healthy older adults (age > 50 years) were recruited using a convenience sampling technique, by advertising the study using flyers, contacting subjects from previous studies, and by word of mouth (Table 1). Informed consent was acquired prior to data collection (IRB No. 20–013160). A homogeneous age distribution was achieved by balancing the number of subjects aged 50 to 64 with those aged 65 and older across both genders. Individuals exhibiting a body mass index (BMI) exceeding 35, encountering difficulties in performing routine activities of daily living, diagnosed with spinal pathologies or neuromuscular disorders, dependent on assistive walking devices, or presenting with a FRAIL score (Morley et al., 2012) of 3 or above were excluded. None of the enrolled subjects had a history of falls.

The subjects were studied while walking on a 10 m long walkway. Retro-reflective markers were attached to the right and the left heels (RHEE and LHEE) and the right and the left anterior and posterior superior iliac spines (RASI, LASI, RPSI, and LPSI) (Fig. 1 (a)). Marker trajectory data were captured at 120 Hz using a 14-camera real-time motion capture system (Raptor 12HS, Motion Analysis, Rohnert Park, CA). Each subject provided 5 walking trials. The first and last two strides were omitted to account for gait initiation and termination, resulting in an average of 13 strides being analyzed. Any instability during the trials such as falls, stumbles, trips, or loss of balance were recorded.

The RPSI, LPSI, RASI, and LASI marker trajectories were averaged to obtain a surrogate CoM (Granata and Lockhart, 2008), and the RHEE and LHEE marker trajectories were averaged to obtain a surrogate center of pressure trajectory (CoP) (Granata and Lockhart, 2008) (Fig. 1 (b)). Maintaining medio lateral stability is complicated, and is required for an upright posture during gait (Fettrow et al., 2019; Patla et al., 1999). Hence, COM movement with respect to COP movement was calculated in the medio-lateral direction and normalized to obtain a bounded quasi periodic system, $y(t)$, such that $\max(\|y(t)\|) = 1$ and $\overline{y(t)} = 0$. The timeseries data from individual trials, $y(t)$, were stitched and filtered for noise (forth order lowpass Butterworth filter with a 10 Hz cutoff frequency (Winter, 2009)) to form an extended data timeseries, $Y(t)$ (Fig. 1 (c)). A phase space representation was created for the system as $m(t) = [Y(t), \dot{Y}(t)]$.

2.2. Time-delayed embedding

Time-delayed embedding facilitates reconstruction of a system's dynamics using measurements in the real world (Takens, 1979). The observed system $m(t)$ was time delayed and the N time delayed copies of the system were stacked to form a higher dimensional system representation:

$$M(t) = [m(t), m(t+\tau), m(t+2\tau), \cdots, m(t+N\tau)]$$

where τ was the time delay, and $N = floor(\frac{k-T}{\tau})$ where k is the total length of the dataset $Y(t)$ and T is the averaged gait cycle duration. This resulted in a higher dimensional system such that $M(t) \in \mathbb{R}^{k\times N}$.

Time-delayed embedding has been performed by selecting a value for the time delay and the embedding dimension. Several methods to obtain the time-delay values and embedding dimension exist, ranging from a commonly used literature value (Bisi et al., 2018; Caronni et al., 2020; Lockhart and Liu, 2008; Tamburini et al., 2018) to finding the optimum value for each individual system (Bhat et al., 2021; Dingwell and Cusumano, 2000; Kennel et al., 1992; Wallot and Mønster, 2018). In this study, the optimum value for the time delay was obtained by employing the Average Mutual Information (AMI) technique on the observed system $m(t)$ using a MATLAB function (Leontitsis, 2020). The value of τ was selected to be the first minima of the AMI curve, thereby resulting in the least amount of information lost due to the time delay.

Three methods were used to obtain the embedding dimension and to obtain the FT metrics:

1 Method A (FNN+numerical approximation):

The value of N was set to 10 for every subject such that the number of strides analyzed for each subject was the same. The embedding dimension (n) was calculated for the time-delayed system by applying the false nearest neighbor (FNN) approach (Kennel et al., 1992). Using Rosenstein et al's algorithm, the short-term (over 0 to 0.5 gait cycles) and long-term (over 6 to 8 gait cycles) LE were estimated (Rosenstein et al., 1993). Rosenstein et al's algorithm does not provide the FM values.

2 Method B (FNN+SAT calculation):

The embedding dimension (n) was calculated as described in Method A. The Floquet transition matrix (FTM) for the system was calculated semi-analytically as follows:

$$\Phi(T) = M(T)M^{-1}(0)$$

where T is the principal time period (average gait cycle duration). The eigen values of the FTM were the FM. The FTM can also be expressed as an exponential (Bhat et al., 2020; Teschl, 2012).

$$\Phi(T) = e^{RT}$$

The eigen values of R are the Floquet exponents. LE are defined as the real part of the Floquet exponents.

$$LE = real(eig(R))$$

Using this equations-based approach instead of numerical approximations leads to the calculation of both the FM and the long-term LE for a given dynamical system.

3 Method C (SVD+SAT calculation):

All collected data was utilized instead of limiting the number of gait cycles. The embedding dimension was calculated for the time-delayed system by the dimensionality reduction procedure using SVD $M(t) = U(t)S(t)V(t)$ as described by Broomhead et al. (Broomhead and King, 1986). The singular value matrix ($\Sigma(t)$) for the high dimensional system consists of singular values ($S_{i,j}$) in descending order. The most significant normalized singular values ($\frac{S_{i,j}}{\sum S_{i,j}}*100 > 1$) were selected such that the reduced system retained 99 % of the system dynamics, also known as the rank of the matrix. The matrix rank was the embedding dimension (n). The values of FM and long-term LE were calculated like Method B.

Table 1
Subject demographics.

	Values (Mean ± Std)
N (Male/Female)	33 (15/18)
Age (years)	64 ± 10
Height (m)	1.7 ± 9.2
Weight (kg)	77.8 ± 12.9

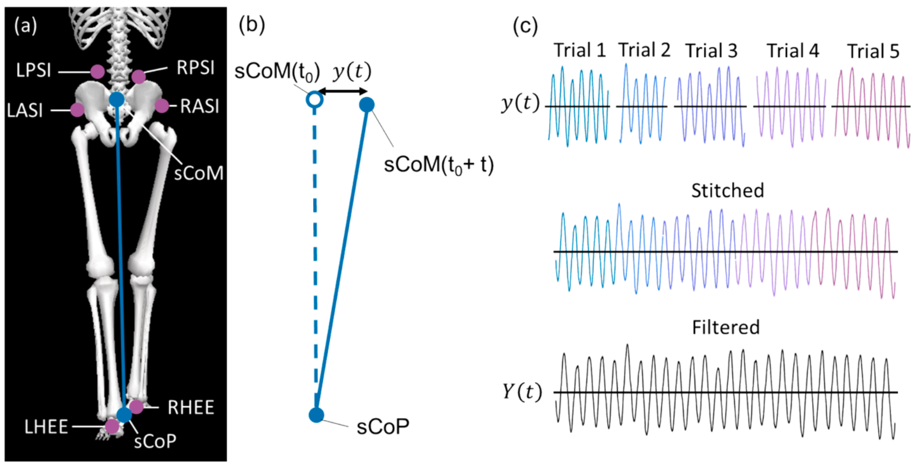

Fig. 1. (a) Frontal plane representation of the subject in Visual3d during gait, (b) representation of the calculation of $y(t)$ using the position of the CoM relative to the CoP, (c) mean normalized medio-lateral trajectory of the CoM for 5 trials and the stitched and filtered longer data timeseries $Y(t)$. The movement of the CoM was similar for each trial. LPSI: left superior posterior iliac spine; RPSI: right superior posterior iliac spine; LASI: left superior anterior iliac spines; RASI: right superior anterior iliac spines; CoM: surrogate center of mass; CoP: surrogate center of pressure.

2.3. Statistics

A within-subjects ANOVA was used to assess if there were any differences between the short-term and long-term LE from Method A, and the long term LE from Methods B, and C. Post-hoc multiple comparisons were made using the Sture Holm method (Holm, 1979). A separate within-subjects ANOVA was used to determine if there were any differences between the FM calculated using Methods B and C. Statistical significance was set at $p < 0.05$.

3. Results

All subjects completed all walking trails without any falls, stumbles, trips, or loss of balance. The LE were significantly different. Post hoc tests showed that the long-term LE calculated using Method A were similar to the long-term LE calculated using the Method B (Fig. 2). All other pairings of the maximal LE were significantly different. The short-term LE from Method A (mean ± std.: 2.59 ± 0.28) indicated that the gait for all the subjects was chaotic. The long-term LE from Method A (0.03 ± 0.14) and the long-term LE from Method B (mean ± std.: −0.09 ± 0.45) indicated that the gait of about half of the subjects was chaotic. The long-term LE from Method C (−0.30 ± 0.29) indicated that most of the subjects had a non-chaotic gait (Fig. 2).

The FM from both Methods B (0.83 ± 0.41) and C (0.68 ± 0.18) were significantly different. The values from Method B indicated that the gait for over 30 % of the subjects was unstable (Fig. 3).

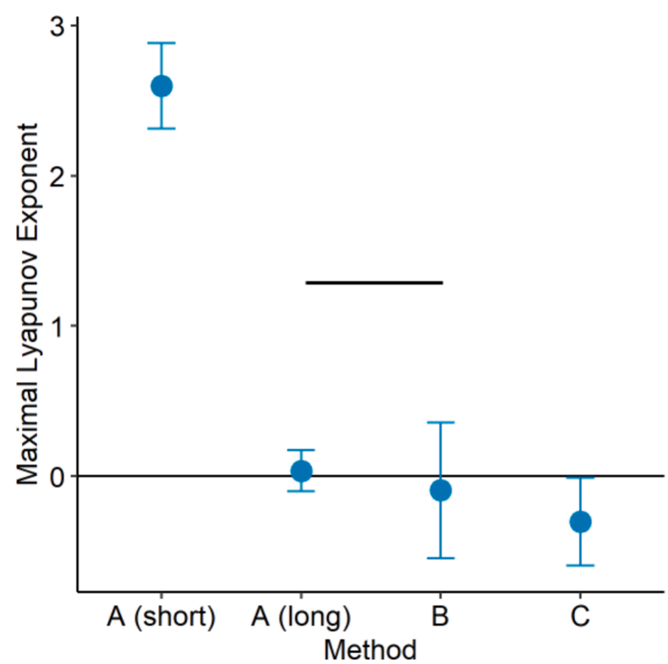

Fig. 2. Comparison between the maximum Lyapunov exponents (LE) calculated using different methods. The means and standard deviations are plotted using the point and vertical lines. The horizontal black line indicates LE=0. Values above the black line indicate a chaotic system. Only the long-term LE for Method A and the long-term LE for Method B are statistically similar (denoted by a short horizontal line above them).

4. Discussion

The current study assessed dynamic stability during gait using three different methods for applying FT. The SAT value of LE and FM obtained using SVD for embedding dimensions and the equations from the FT (Method C) yielded results that differed from the other two methods. The values from Method C described the subjects' gait much closer to their actual performance. Specifically, none of the subjects were unstable during the testing.

SVD has been used to analyze dynamic system performance embedded in noisy data (Chungyong and Williams, 1995; Shin et al., 1999). SVD's application for finding the embedding dimension is well documented (Broomhead and King, 1986). Marmelat et al. used shorter walking trials of patients with Parkinson's disease and reported that stitching them did not affect the gait variability measures of interest, but did induce noise between the time delayed values of the gait variable (Marmelat et al., 2018). Therefore, using SVD to study gait system dynamics using shorter walking trials ensures reliability for interpretation of these measures.

The ranges of the short-term as well as long-term LE reported in this study were similar those reported previously by Mehdizadeh for a similar age group (Mehdizadeh, 2018). The current study also demonstrated that using the Rosenstein et al.'s algorithm yielded similar values of long-term LE as the SAT solution. As per FT, a positive LE describes the presence of chaos in the system, which has been correlated with fall frequency (Lockhart and Liu, 2008; Reynard et al., 2014). The subjects in the current study were community dwelling older adults with no history of falls. Their gait exhibited perturbation tolerance which means

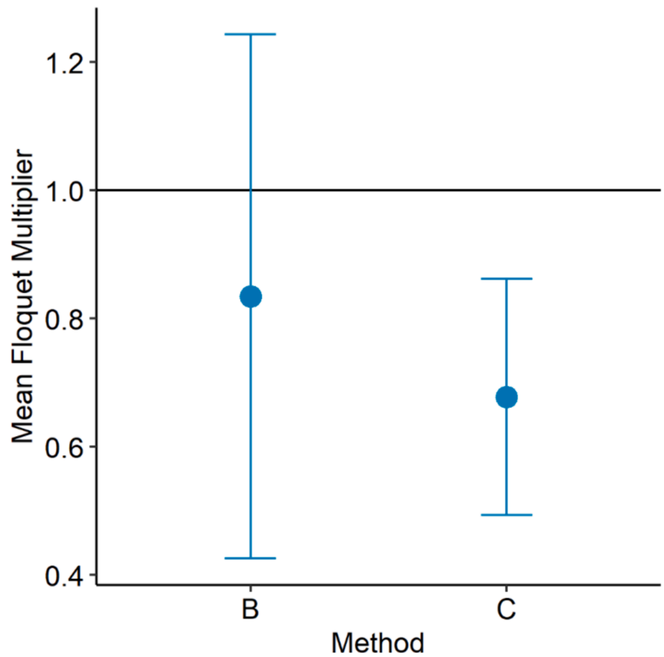

Fig. 3. Comparison between the mean Floquet multipliers (FM) calculated using the Methods B and C. The method means and standard deviations are plotted using the point and vertical lines. The horizontal black line indicates FM=1. Values above the black line indicate an unstable system. The Methods B and C yielded significantly different FM.

that the LE should be non-positive. The FM values derived by Method B indicated that about 30 % of the subjects had an unstable gait which does not match the subjects' actual gait. In contrast, Method C showed that all the subjects had a stable gait. Hence, utilization of the SVD approach for calculating the embedding dimension of shorter walking trials should be explored further.

This study has certain limitations. The FT should be applied only to periodic systems whereas gait is considered a quasi-periodic system. Hence, application of FT to gait is an approximation. Gait velocity has been shown to affect measures of gait and balance (Mehdizadeh, 2018) and was not included as a covariate in the current study. The current study mean normalized the COM trajectory to achieve similar effects as normalization with respect to gait velocity.

5. Conclusion

The SVD and SAT approach yielded values of LE and FM that most closely matched the subjects' gait patterns.

CRediT authorship contribution statement

Sandesh G. Bhat: Writing – review & editing, Writing – original draft, Resources, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. **Kenton R. Kaufman:** Writing – review & editing, Writing – original draft, Supervision, Resources, Project administration, Methodology, Funding acquisition, Formal analysis, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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